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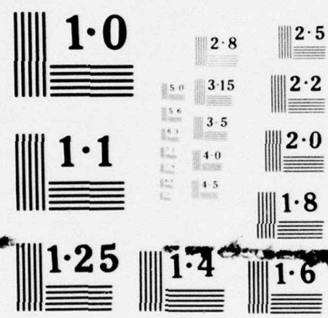

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by

(10) Mahesh Chandra
Nozer D. Singpurwalla

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In this note we consider some test statistics based on the sample distribution function for testing the null hypothesis that a random sample belongs to a Weibull distribution with unknown scale and shape parameters. A foundation for testing such a hypothesis is provided by the fact that the logarithm of a Weibull random variable has an extreme value distribution with a location and a scale parameter, and by some recent results of Durbin (1973) and of Serfling and Wood (1976). These results pertain to the weak convergence of an associated "empirical" stochastic process, under the null hypothesis. The asymptotic distribution of the empirical process serves as a basis for Monte Carlo studies for determining the appropriate critical points of the test statistics. We shall give some results on comparing the power of our tests and a test due to Mann, Scheuer, and Fertig (1973).

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1. Introduction

The two-parameter Weibull distribution has found many applications in the engineering and in the biological sciences. For instance, it has been used by Cook, Doll and Fellingham (1969) and by Doll (1971), to describe the observed age distribution of many human cancers. Its use for describing failures of electrical and mechanical components is well documented in the reliability literature.

In this note we address ourselves to a fundamental problem involving any application of the Weibull distribution. We wish to test the null hypothesis that a given random sample belongs to a Weibull distribution with unknown parameters. Of the several methods for testing "goodness of fit," those based on the sample distribution function happen to be the most popular. We shall present tables of critical values for testing the null hypothesis in question, and also give some results comparing the power of our tests and a test due to Mann, Scheuer and Fertig (1973).

A foundation for developing our tables of critical values is the recently given theory by Durbin (1973), and by Serfling and Wood (1976) on the weak convergence of an "empirical" stochastic process. This

stochastic process is based on the sample distribution function and estimates of the unknown parameters. The statistics that we discuss can be represented as well-behaved functionals of this empirical process. Thus, the asymptotic distributions of the relevant test statistics can be obtained as the distributions of the corresponding functionals of the limiting process. The above ideas will be made clear in the following text.

2. Preliminaries

The two-parameter Weibull distribution is given by

$$\begin{aligned} F(t) &= 1 - \exp\left[-\left(\frac{t}{\delta}\right)^{\beta}\right], \quad & t \geq 0 \\ &= 0 \quad &, \text{ otherwise;} \end{aligned} \tag{2.1}$$

the scale parameter δ and the shape parameter β are both assumed to be positive.

If we make the transformation $X = -\ell n\ T$, where T has a two-parameter Weibull distribution, then the distribution of X is called the extreme value distribution. It is given by

$$F(x) = \exp\left(-\exp - \left(\frac{x-a}{b}\right)\right), \tag{2.2}$$

where $a = -\ell n\ \delta$ and $b = \frac{1}{\beta}$. We note that a and b are, respectively, the location and the scale parameters of the extreme value distribution.

The tests that we discuss in this paper are based on the extreme value distribution. To make a test of fit to the Weibull distribution we shall first take the negative of the natural logarithms of the supposed Weibull data. Thus, we wish to consider the case of testing whether the distribution of a random sample $X_1, X_2, \ldots, X_n$ is an extreme value distribution with unknown location parameter a and unknown scale parameter b. Specifically, we wish to test the "null hypothesis"

$$H_0 : \quad F(x) = \exp\left[-\exp - \left(\frac{x-a}{b}\right)\right],$$

for all x and for some (a,b) .

When a and b are specified, then H_0 is "simple," and our test reduces to testing the hypothesis that the independent random variables

$$G\left[\frac{X_i-a}{b}\right] = \exp\left[-\exp - \left(\frac{X_i-a}{b}\right)\right], \qquad 1 \le i \le n ,$$

have a common uniform $(0,1)$ distribution. The Kolmogorov-Smirnov test is based on the statistic

$$n^{1/2} \sup_{0\le t\le 1} |G_n(t) - t| , \tag{2.3}$$

where

$$G_n(t) = \frac{1}{n} \sum_{i=1}^{n} I\left(G\left[\frac{X_i-a}{b}\right] \le t\right), \qquad 0 \le t \le 1 , \tag{2.4}$$

where $I(E)$ denotes the indicator of the event E . Under the null hypothesis, the "empirical" stochastic process

$$W_n(t) = n^{1/2} [G_n(t) - t] , \qquad 0 \le t \le 1 \tag{2.5}$$

satisfies

$$W_n \xrightarrow{d} W^0 \quad \text{in} \quad \mathcal{D}[0,1] , \tag{2.6}$$

where $\xrightarrow{d}$ denotes convergence in distribution and W^0 denotes the Gaussian process determined by

$$E[W^0(t)] = 0 , \qquad 0 \le t \le 1$$

and

$$E[W^0(s)W^0(t)] = \min(s,t) - st , \quad 0 \le s,\ t \le 1 .$$

$\mathcal{D}[0,1]$ denotes the space of functions on $[0,1]$ which are right-continuous and have left-hand limits.

In the following section we present some results on an analogous test statistic for the case H_0 composite. These results will serve as a basis for developing the tables of critical values.

3. The Convergence Theorem and the Test Statistic

When a and b are not specified, that is, when H_0 is composite, we consider an analogous approach based on $(\hat{a}_n, \hat{b}_n)$, the maximum likelihood estimators of (a,b). We set

$$Y_{n,i} = \frac{X_i - \hat{a}_n}{\hat{b}_n}, \qquad 1 \leq i \leq n,$$

and analogous to G_n and W_n we define

$$H_n(t) = \frac{1}{n} \sum_{i=1}^{n} I[G(Y_{n,i}) \leq t], \qquad 0 \leq t \leq 1$$

and

$$V_n(t) = n^{1/2} [H_n(t) - t], \qquad 0 \leq t \leq 1.$$

Our theorem pertains to the "empirical" stochastic process $V_n(t)$, and is analogous to the result given by Equation (2.6). However, before stating the convergence theorem, we will have to introduce the following notation given in Durbin (1973), and verify that his assumptions (conditions) are satisfied.

Let us denote by θ the vector $[a,b]'$, and let θ_0 be any conveniently chosen value of θ. We state below a verification of the required conditions.

Condition A: The distribution $G(x,\theta_0)$ has a density $f(x,\theta_0)$ such that, for almost all x, the vector $\partial \log f(x,\theta_0)/\partial\theta_0$ exists, and satisfies

$$E\left(\frac{\partial \log f(x,\theta_0)}{\partial\theta_0}, \frac{\partial \log f(x,\theta_0)}{\partial\theta_0'}\right) = J,$$

where J is finite and positive definite.

Condition B: Let $\hat{\theta}_n$ be the maximum likelihood estimator of θ ; that is, $\hat{\theta}_n = [\hat{a}_n, \hat{b}_n]'$. Then, it is well known (cf. Cramer (1946)] that

$$n^{1/2}(\hat{\theta}_n - \theta_0) = \frac{1}{n^{1/2}} \mathcal{J}^{-1} \sum_{i=1}^{n} \frac{\partial \log f(x_i, \theta_0)}{\partial \theta_0} + \varepsilon_n ,$$

where $\varepsilon_n \to 0$, in probability.

Condition C: Let N be the closure of a neighborhood of θ_0 . Let $g(t,\theta) = \partial G(x,\theta)/\partial\theta$ when this is expressed as a function of t by means of the transformation $t = G(x,\theta)$; let $g(t) = g(t,\theta_0)$. The vector function $g(t,\theta)$ is continuous in (θ,t) for all $\theta \varepsilon N$, and $0 \leq t \leq 1$.

Theorem 3.1: By virtue of Conditions A, B, and C, the "empirical" process V_n determined by the extreme value distribution $G\left[\frac{x-a_n}{b_n}\right]$, with (a_n, b_n) the maximum likelihood estimators, is such that

$$V_n \xrightarrow{d} V^0 \quad \text{in} \quad \mathcal{D}[0,1] ,$$

where V^0 is a Gaussian process determined by

$$E[V^0(t)] = 0 , \qquad 0 \leq t \leq 1$$

and

$$E[V^0(s)V^0(t)] = \min(s,t) - st - g(s)'\mathcal{J}^{-1}g(t) , \qquad 0 \leq s, t \leq 1 . \quad (3.1)$$

Proof: Follows from Durbin (1973). //

If we choose $\theta_0 = [0,1]'$, then it can be verified that $g(t) = [t\log t, -t\log t \log(-\log t)]$, and that

$$\mathcal{J}^{-1} = \begin{vmatrix} 1.10867 & 0.257 \\ 0.257 & 0.60793 \end{vmatrix} ,$$

[cf. Johnson and Kotz (1970), p. 282]. Substituting the above into (3.1) we have the covariance of our Gaussian process

$$\begin{aligned} E[V^0(s)V^0(t)] = \min(s,t) &- st - 1.108(s\log s)(t\log t) \\ &+ 0.257(s\log s)\big(t\log t \log(-\log t)\big) \\ &+ 0.257\big(s\log s \log(-\log s)(t\log t)\big) \\ &- 0.60793\big(s\log s \log(-\log s)\ t\log t \log(-\log t)\big) \ , \quad 0 \le s,\ t \le 1 \ . \end{aligned} \tag{3.2}$$

The statistics of interest in connection with H_0 are:

(i) the one-sided Kolmogorov statistic

$$D_n^+ = \sup_{0 \le t \le 1} V_n(t) \ , \tag{3.3}$$

$$D_n^- = -\inf_{0 \le t \le 1} V_n(t) \ , \tag{3.4}$$

(ii) the Kolmogorov-Smirnov statistic

$$D_n = \max(D_n^+, D_n^-) \ , \tag{3.5}$$

(iii) the Kupier statistic

$$V_n = D_n^+ + D_n^- \ , \tag{3.6}$$

(iv) the Cramer-Von Mises statistic

$$W_n^2 = \int_0^1 V_n^2(t)dt \ , \tag{3.7}$$

(v) the Watson statistic

$$U_n^2 = \int_0^1 V_n^2(t)dt - \left[\int_0^1 V_n(t)dt\right]^2 \ , \tag{3.8}$$

and

(vi) the Anderson-Darling statistic

$$A_n^2 = \int_0^1 \frac{V_n^2(t)}{t(1-t)} dt \ . \tag{3.9}$$

Using the fact that if $h\left(V_n(t)\right)$ is a function of $V_n(t)$ that is continuous with respect to the Skorkhod metric on $D[0,1]$, the limit laws of D_n^+, D_n^-, D_n, V_n, W_n^2, U_n^2, and A_n^2 under H_0 are given, respectively, by the laws of the random variables

$$D^+ = \sup_{0\le t\le 1} V^0(t) , \tag{3.10}$$

$$D^- = -\inf_{0\le t\le 1} V^0(t) , \tag{3.11}$$

$$D = \max(D^+, D^-) , \tag{3.12}$$

$$V = D^+ + D^- , \tag{3.13}$$

$$W^2 = \int_0^1 \left(V^0(t)\right)^2 dt , \tag{3.14}$$

$$U^2 = \int_0^1 \left(V^0(t)dt\right)^2 - \left[\int_0^1 V^0(t)dt\right]^2 , \tag{3.15}$$

and

$$A^2 = \lim_{\varepsilon\to 0} \int_{0+\varepsilon}^{1-\varepsilon} \frac{\left(V^0(t)\right)^2}{t(1-t)} dt . \tag{3.16}$$

The above results follow as a consequence of the continuous mapping theorem of Billingsley (1968). They provide a basis for Monte Carlo studies of the null hypothesis asymptotic distributions of the statistics discussed above.

4. Sampling Distributions of the Approximate Test Statistics

Monte Carlo methods were used to simulate the distribution of the limiting random variables given in Equations (3.10) through (3.16). Following Serfling and Wood (1976) we approximate the Gaussian process V^0 by its finite-dimensional distributions, corresponding to an evaluation of the process at 29, 99, and 119 equally-spaced points in the unit

interval. One thousand multivariate normal random vectors with the covariance given by Equation (3.2) were generated using a program from the International Mathematical and Statistical Library. The empirical distributions of the supremum, the infimum, and the difference between the supremum and the infimum of the resulting multivariate normal vectors were then tabulated, thus approximating the limit laws of D_n^+ , D_n^- , D_n , and V_n . Since the differences in the observed quantiles corresponding to the finite-dimensional distributions of V^0 at 29, 99, and 119 equally-spaced points diminished, the approximating procedure was terminated at 119 equally-spaced points. The asymptotic distributions of W^2 , U^2 , and A^2 were obtained by using numerical integration techniques. For this we used Subroutine QSF from the IBM Scientific Subroutine Package. The various sample quantiles for the generated frequency distributions are shown in Table 4.1.

5. The Mann-Scheuer-Fertig (MSF) Test

The only other known procedure for testing goodness of fit for the Weibull that is not based on the empirical distribution function is a test proposed by Mann, Scheuer, and Fertig (1973).

The MSF test is based on a statistic S , and can be used for censored as well as uncensored samples. However, the percentage points of S and certain quantities that are used in calculating S are available only for sample sizes of up to 25. However, along with a modification given by Stephens (1977), the test statistics we discuss can be used for any sample size.

For a sample of size n , censored at m , the statistic S is defined as

$$S = \frac{\sum_{i=[m/2]+1}^{m-1} (X_{i+1} - X_i) / [E(Y_{i+1}) - E(Y_i)]}{\sum_{i=1}^{m-1} (X_{i+1} - X_i) / [E(Y_{i+1}) - E(Y_i)]} ,$$

where $Y_i = \frac{x_i - a}{b}$ and $[r]$ denotes the greatest integer contained in r. Mann, Scheuer, and Fertig give percentage points of S and the values of the quantities $[E(Y_{i+1}) - E(Y_i)]$ for samples of size 3 to 25.

6. Power Comparisons

In order to evaluate the effectiveness of the tests discussed before, we evaluate their power, against the lognormal distribution as an alternative. The lognormal distribution is chosen because it appears to be a natural competitor to a Weibull distribution. The power comparisons were made numerically. For this random samples of size 20, 25, and 30, respectively, were generated from a lognormal (normal) distribution with parameters -0.5 (mean) and 1.00 (variance), respectively.

Maximum likelihood estimators of the parameters a and b of the extreme value distribution were obtained by numerically solving the following equations simultaneously:

$$\hat{b} = \sum_j X_j/n - \left[\sum_j X_j \exp(-X_j/\hat{b})\right]\left[\sum_j \exp(-X_j/\hat{b})\right]^{-1} \tag{6.1}$$

and

$$\hat{a} = -\hat{b}\log\left[\sum_j \exp(-X_j/\hat{b})/n\right]. \tag{6.2}$$

The results of our power comparisons are shown in Tables 6.1, 6.2, and 6.3, and these are based on 1000 replicates. Based on this limited experiment, it appears that for samples of sizes 20 and 25, the MSF test has better power. For samples of size 30, the MSF test could not be used, and the Anderson-Darling test appears to have better power.

7. Concluding Remarks

After finishing the work on this report we were informed that Stephens (1977) has also obtained asymptotic percentage points for the statistics W^2, U^2, and A^2. Stephens also gives a necessary modification so as to use these statistics for a finite sample sizes. Even though

TABLE 6.1

POWER COMPARISON: WEIBULL vs. LOGNORMAL
SAMPLE SIZE n = 20

Level of Significance	Kolmogorov-Smirnov			Kupier	Cramer von Mises	Watson	Anderson Darling	Mann Scheuer Fertig
	D^+	D^-	D	V	W^2	U^2	A^2	S
0.01	0.008	0.101	0.075	0.102	0.080	0.082	0.100	0.105
0.025	0.023	0.167	0.114	0.131	0.144	0.140	0.177	Not Available
0.05	0.046	0.236	0.171	0.209	0.219	0.211	0.238	0.265
0.10	0.084	0.418	0.249	0.303	0.322	0.321	0.354	0.432

TABLE 6.2

POWER COMPARISON: WEIBULL vs. LOGNORMAL
SAMPLE SIZE n = 25

Level of Significance	Kolmogorov-Smirnov			Kupier	Cramer von Mises	Watson	Anderson Darling	Mann Scheuer Fertig
	D^+	D^-	D	V	W^2	U^2	A^2	S
0.01	0.019	0.145	0.121	0.141	0.131	0.132	0.166	0.153
0.025	0.034	0.196	0.155	0.189	0.196	0.190	0.240	Not Available
0.05	0.072	0.263	0.206	0.255	0.279	0.264	0.310	0.389
0.10	0.116	0.457	0.286	0.360	0.364	0.365	0.411	0.533

TABLE 6.3

POWER COMPARISON: WEIBULL vs. LOGNORMAL
SAMPLE SIZE n = 30

Level of Significance	Kolmogorov-Smirnov			Kupier	Cramer von Mises	Watson	Anderson Darling	Mann Scheuer Fertig
	D^+	D^-	D	V	W^2	U^2	A^2	S
0.01	0.021	0.173	0.137	0.161	0.151	0.145	0.183	Not Available
0.025	0.042	0.252	0.185	0.210	0.231	0.219	0.286	Not Available
0.05	0.082	0.331	0.256	0.296	0.333	0.319	0.369	Not Available
0.10	0.132	0.521	0.355	0.412	0.421	0.418	0.475	Not Available

our approach is different, it is encouraging to note that our results seem to be in good agreement with those of Stephens. A comparison of the asymptotic points we obtained with those of Stephens is given in Table 7.1. Stephens has made no power comparisons, and since our results agree quite well with his, we conclude that our power comparisons remain valid.

ACKNOWLEDGMENT

We are grateful to Professor Robert J. Serfling of the Florida State University, who was kind enough to provide us with a preprint of his paper. Our work is motivated by Section 4 of his paper.

TABLE 7.1

COMPARISON OF UPPER TAIL PERCENTAGE POINTS OF W^2, U^2, AND A^2 STATISTICS

Statistic	$\alpha = 0.75$		$\alpha = 0.90$		$\alpha = 0.95$		$\alpha = 0.975$		$\alpha = 0.99$	
	Table 4.1	Stephens	Table 4.1	Stephens	Table 4.1	Stephens	Table 4.1	Stephens	Table 4.1	Stephens
W^2	0.073	0.073	0.105	0.102	0.123	0.124	0.147	0.146	0.175	0.175
U^2	0.069	0.070	0.098	0.097	0.117	0.117	0.140	0.138	0.164	0.165
A^2	0.457	0.474	0.623	0.637	0.746	0.757	0.849	0.877	0.991	1.038

REFERENCES

BILLINGSLEY, PATRICK (1968). Convergence of Probability Measures. Wiley, New York.

COOK, P., R. DOLL and S. A. FELLINGHAM (1969). A mathematical model for the age distribution of cancer in man. Internat. J. Cancer **4** 93-112.

CRAMER, H. (1946). Mathematical Methods of Statistics. Princeton University Press, Princeton, New Jersey.

DOLL, R. (1971). The age distribution of cancer: implications for models of carcinogens. J. Roy. Statist. Soc. Ser. A **134** 133-166.

DURBIN, J. (1973). Weak convergence of the sample distribution function when parameters are estimated. Ann. Statist. **1** 279-290.

JOHNSON, N. L. and S. KOTZ (1970). Continuous Univariate Distributions, **1**, Houghton-Mifflin Company, Boston.

MANN, NANCY R., E. M. SCHEUER and K. W. FERTIG (1973). A new goodness of fit test for the two parameter Weibull or extreme value distribution with unknown parameters. Comm. in Statistics **2** (5) 383-400.

SERFLING, R. J. and C. L. WOOD (1976). On null hypothesis limiting distributions of Kolmogorov-Smirnov type statistics with estimated location and scale parameters. Technical report, Florida State University, Tallahassee, Florida.

STEPHENS, M. A. (1977). Goodness-of-fit for the extreme value distribution. Technical Report No. 247, Department of Statistics, Stanford University, Stanford, California.

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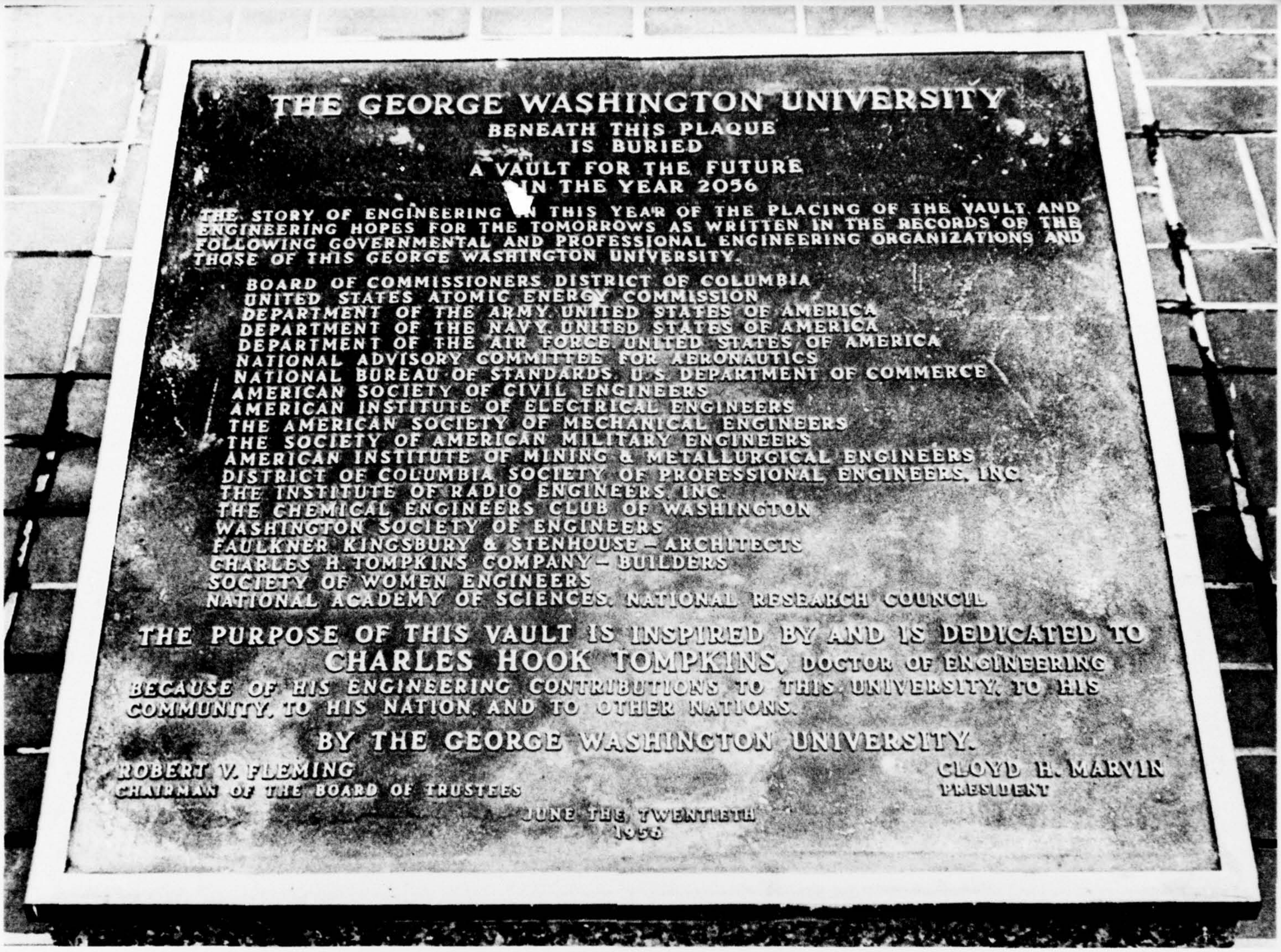

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